Period 3, 8/26/24

$$\lim_{a\to 0}\frac{1-\cos a}{a}=\lim_{a\to 0}\frac{\cos a-1}{a}=0$$

e) $\lim\limits_{x\to 0}\frac{1-\cos 2x}{4x}=\lim\limits_{x\to 0}\frac{1-\cos 2x}{2\cdot 2x}$

$a=2x$

$$\lim_{x\to 0}\frac{1}{2}\cdot\boxed{\frac{(1-\cos 2x)}{2x}}=\lim_{a\to 0}\frac{1}{2}\cdot\boxed{\frac{(1-\cos a)}{a}}$$

0 0

$\frac{1}{2}\cdot 0=0$

2. Given $\frac{\cos x-1}{x}\le h(x)\le e^x-1$, find $\lim\limits_{x\to 0}h(x)$.

$e^0-1=1-1=0$

$$\lim_{x\to 0}\frac{\cos x-1}{x}\le\lim_{x\to 0}h(x)\le\lim_{x\to 0}e^x-1$$

$0\le\lim\limits_{x\to 0}h(x)\le 0\Rightarrow$ Squeeze Theorem

e)

$$\lim_{x\to 0}\frac{\frac{1}{3+x}-\frac{1}{3}}{x}$$

$$\frac{3\cdot 1}{3\cdot(3+x)}-\frac{1(3+x)}{3(3+x)}=\frac{3}{3(3+x)}-\frac{3+x}{3(3+x)}$$

$$\frac{\cancel{3}-\cancel{3}-x}{3(3+x)}$$

$$\lim_{x\to 0}\frac{\frac{-x}{3(3+x)}}{\frac{x}{1}}$$

$$\lim_{x\to 0}\frac{-\cancel{x}}{3(3+x)}\cdot\frac{1}{\cancel{x}}=\lim_{x\to 0}\frac{-1}{3(3+x)}=\frac{-1}{3(3+0)}=\frac{-1}{9}$$

$$\lim_{a\to 0}\frac{\sin a}{a}=\lim_{a\to 0}\frac{a}{\sin a}=1$$

$5\sin x\neq\sin 5x$

c) $\lim\limits_{x\to 0}\frac{4x}{\sin(5x)}$

$$\lim_{x\to 0}\frac{5\cdot 4x}{5\cdot\sin(5x)}$$

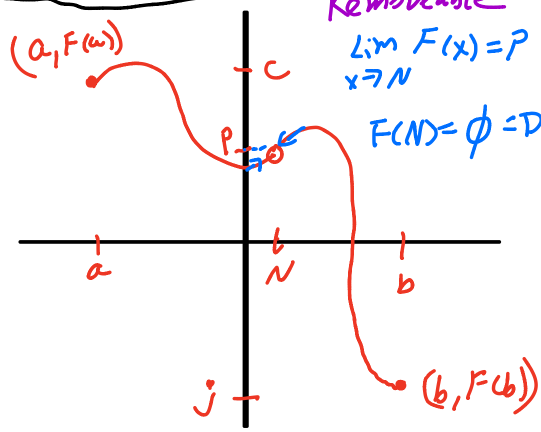
$$\lim_{x\to 0}\frac{4\cdot 5x}{5\cdot\sin 5x}$$

$a=5x$

$$\lim_{x\to 0}\frac{4}{5}\cdot 1=\frac{4}{5}$$

Continuity at $x=k$

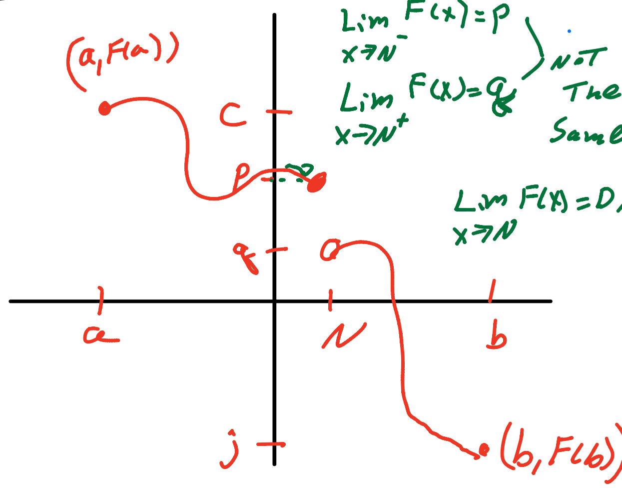
1. $\lim_{x \to k} F(x)$ must exist
2. $F(k)$ must exist
3. $\lim_{x \to k} F(x) = F(k)$



Removeable

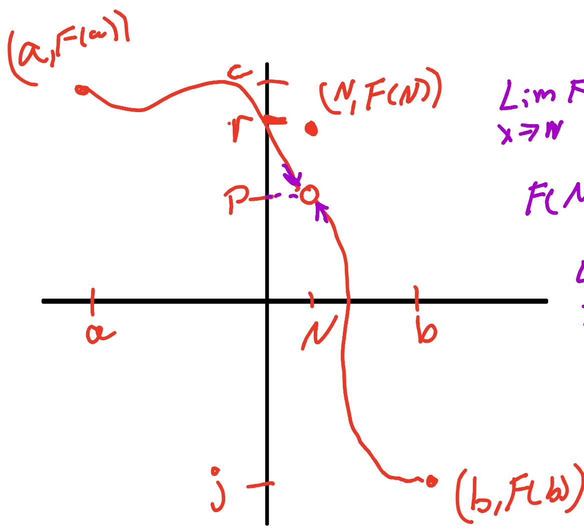
$\lim_{x \to N} F(x) = P$

$F(N) = \emptyset = DNE$



$\left.\begin{array}{l} \lim_{x \to N^-} F(x) = p \\ \lim_{x \to N^+} F(x) = q \end{array}\right\}$ Not The Same

$\lim_{x \to N} F(x) = DNE$



$\lim_{x \to N} F(x) = P$

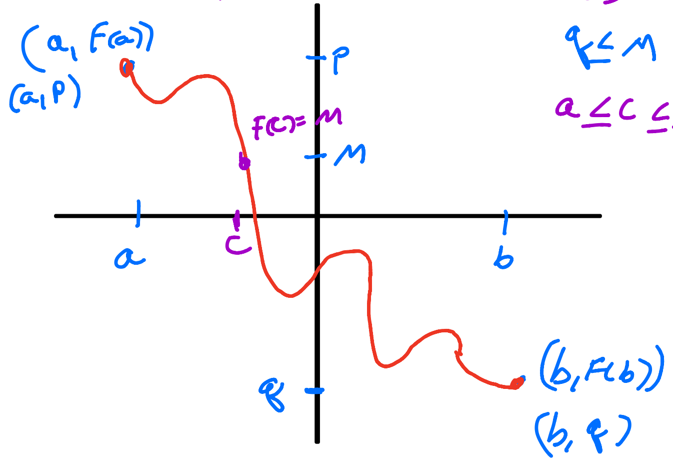
$F(N) = r$

$\lim_{x \to N} F(x) = F(N)$

$p \neq r$

Removeable

IVT must be continious



$q \le M \le P$

$a \le c \le b$

Example 2

Discuss the continuity of each function.

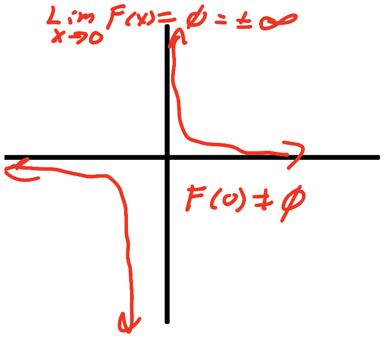
$$f(x) = \frac{1}{x}$$

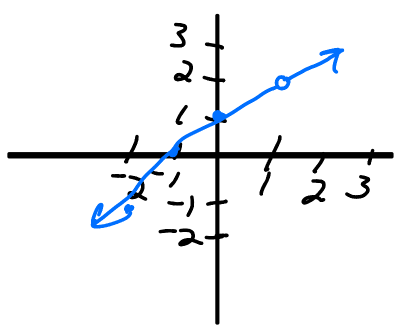
$$g(x) = \frac{x^2 - 1}{x - 1} = \frac{(x-1)(x+1)}{(x-1)}$$

$$h(x) = \begin{cases} x + 1, & x \le 0 \\ x^2 + 1, & x > 0 \end{cases}$$

Not continious at $x = 0$

$\lim_{x \to 0} F(x) = \phi = \pm\infty$

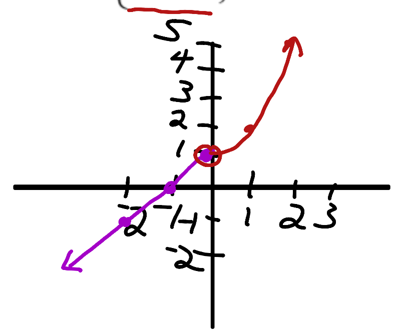




$\lim_{x \to 1} F(x) = 2$

$F(1) = \frac{0}{0} = \phi = DNE$

Not continious at $x = 1$



$\lim_{x \to 0^+} h(x) = 1$

$\lim_{x \to 0^-} h(x) = 1$

$\lim_{x \to 0} h(x) = 1$

$h(0) = 1$

$\lim_{x \to 0} h(x) = h(0)$

$1 = 1$

a) $g(x) = \begin{cases} x^2 + 7, & x \geq 1 \\ x + a, & x < 1 \end{cases}$

$\lim_{x \to 1^+} x^2+7 = 1^2+7 = 8$

$\lim_{x \to 1^-} x+a = 1+a$

MUST Equal Each other

$8 = 1+a$

$7 = a$

b) $h(x) = \begin{cases} \dfrac{x^4 - 1}{x - 1}, & x \neq 1 \\ 4 = a, & x = 1 \end{cases}$

$$\frac{x^4-1}{x-1} = \frac{(x^2-1)(x^2+1)}{x-1} = \frac{(x-1)(x+1)(x^2+1)}{x-1}$$

$$\lim_{x \to 1} \frac{\cancel{(x-1)}(x+1)(x^2+1)}{\cancel{(x-1)}} = (1+1)(1^2+1) = 4$$

$(2)(2) = 4$

$F(1) = 4 = a$

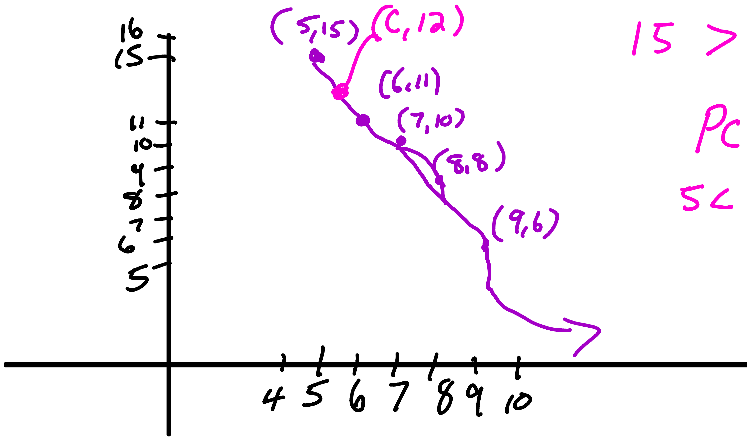
t	5	6	7	8	9
$P(t)$	15	11	10	8	6

Selected values of the function $P(t)$ are given in the table above. $P(t)$ is a continuous $\boxed{\text{decreasing}}$ function. Explain why there must be a value s for $5 < s < 9$ such that $P(s) = 12$.

one value

$P(5) > 12 > P(6)$

$15 > 12 > 11$



$P(c) = 12$

$5 < c < 6$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	–4	4	2
4	–1	3	6	7

3. The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.